

Two-dimensional phase transition models and $\lambda\phi^4$ field theory

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Abstract

The overview is given of the results obtained recently in the course of renormalization-group (RG) study of two-dimensional (2D) models. RG functions of the two-dimensional n -vector $\lambda\phi^4$ Euclidean field theory are written down up to the five-loop terms and perturbative series are resummed by the Padé-Borel-Leroy techniques. An account for the five-loop term is shown to shift the Wilson fixed point only briefly, leaving it outside the segment formed by the results of the lattice calculations. This is argued to reflect the influence of the non-analytical contribution to the β -function. The evaluation of the critical exponents for $n = 1$, $n = 0$ and $n = -1$ in the five-loop approximation and comparison of the results with known exact values confirm the conclusion that non-analytical contributions are visible in two dimensions. The estimates obtained on the base of pseudo- ϵ expansions originating from the 5-loop 2D RG series are also discussed.

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The field-theoretical renormalization-group (RG) approach proved to be a powerful tool for calculating the critical exponents and other universal quantities of the basic three-dimensional (3D) models of phase transitions. Today, many-loop RG expansions for β -functions (six-loop), critical exponents (seven-loop), higher-order couplings (four-loop), etc. of the 3D $O(n)$ -symmetric, cubic, and some other models are known resulting in high-precision numerical estimates for experimentally accessible quantities [1-7]. The main aim of this paper is to demonstrate how effective (or ineffective) is the field-theoretical RG machinery in two dimensions where i) the RG series are stronger divergent and ii) singular (non-analytic) contributions to RG functions are expected to be larger than for 3D systems.

The Hamiltonian of the model describing the critical behavior of various 2D systems reads:

$$H = \int d^2x \left[\frac{1}{2}(m_0^2 \varphi_\alpha^2 + (\nabla \varphi_\alpha)^2) + \frac{\lambda}{24}(\varphi_\alpha^2)^2 \right], \quad (1)$$

where φ_α is a real n -vector field, m_0^2 is proportional to $T - T_c^{(0)}$, $T_c^{(0)}$ being the mean-field transition temperature.

The β -function and the critical exponents for the model (1) are calculated within the massive theory, with the Green function, the four-point vertex and the ϕ^2 insertion being normalized in a conventional way:

$$\begin{aligned} G_R^{-1}(0, m, g_4) &= m^2, & \frac{\partial G_R^{-1}(p, m, g_4)}{\partial p^2} \Big|_{p^2=0} &= 1, \\ \Gamma_R(0, 0, 0, m, g) &= m^2 g_4, & \Gamma_R^{1,2}(0, 0, m, g_4) &= 1. \end{aligned} \quad (2)$$

Since the four-loop RG expansions at $n = 1$ have been obtained many years ago [1], we are in a position to find corresponding series for arbitrary n and to calculate the five-loop terms. The results of our calculations are as follows [8]:

$$\begin{aligned} \frac{\beta(g)}{2} &= -g + g^2 - \frac{g^3}{(n+8)^2} \left(10.33501055n + 47.67505273 \right) \\ &+ \frac{g^4}{(n+8)^3} \left(5.000275928n^2 + 149.1518586n + 524.3766023 \right) \\ &- \frac{g^5}{(n+8)^4} \left(0.0888429n^3 + 179.69759n^2 + 2611.1548n + 7591.1087 \right) \\ &+ \frac{g^6}{(n+8)^5} \left(-0.00408n^4 + 80.3096n^3 + 5253.56n^2 + 53218.6n + 133972 \right), \end{aligned} \quad (3)$$

$$\gamma^{-1} = 1 - \frac{n+2}{n+8} g + \frac{g^2}{(n+8)^2} (n+2) 3.375628955$$

$$\begin{aligned}
& -\frac{g^3}{(n+8)^3} \left(4.661884772n^2 + 34.41848329n + 50.18942749 \right) \\
& +\frac{g^4}{(n+8)^4} \left(0.3189930n^3 + 71.703302n^2 + 429.42449n + 574.58772 \right) \\
& -\frac{g^5}{(n+8)^5} \left(-0.11970n^4 + 69.379n^3 + 1482.76n^2 + 6953.61n + 8533.16 \right), \tag{4}
\end{aligned}$$

$$\begin{aligned}
\eta = & \frac{g^2}{(n+8)^2} (n+2) 0.9170859698 - \frac{g^3}{(n+8)^2} (n+2) 0.05460897758 \\
& +\frac{g^4}{(n+8)^4} \left(-0.09268446n^3 + 4.0564105n^2 + 29.251167n + 41.535216 \right) \\
& -\frac{g^5}{(n+8)^5} \left(0.07092n^4 + 1.05240n^3 + 57.7615n^2 + 325.329n + 426.896 \right); \tag{5}
\end{aligned}$$

the refined expression for the five-loop contribution to γ^{-1} is taken from [9]. Instead of the renormalized coupling constant g_4 , a rescaled coupling

$$g = \frac{n+8}{24\pi} g_4, \tag{6}$$

is used as an argument in above RG series. This variable is more convenient since it does not go to zero under $n \rightarrow \infty$ but approaches the finite value equal to unity.

To evaluate the Wilson fixed point location g^* and numerical values of the critical exponents, the resummation procedure based on the Borel-Leroy transformation

$$f(x) = \sum_{i=0}^{\infty} c_i x^i = \int_0^{\infty} e^{-t} t^b F(xt) dt, \quad F(y) = \sum_{i=0}^{\infty} \frac{c_i}{(i+b)!} y^i, \tag{7}$$

is used. The analytical extension of the Borel transforms is performed by exploiting relevant Padé approximants [L/M]. In particular, four subsequent diagonal and near-diagonal approximants $[1/1]$, $[2/1]$, $[2/2]$, and $[3/2]$ turn out to lead to numerical estimates for g^* which rapidly converge, via damped oscillations, to the asymptotic values; this is clearly seen from Table 1. These asymptotic values, i. e. the final five-loop RG estimates for g^* are presented in Table 2 for $0 \leq n \leq 8$ (to avoid confusions, let us note that models with $n \geq 2$ possessing no ordered phase are studied here only as polygons for testing the numerical power of the perturbative RG technique). As Table 2 demonstrates, the numbers obtained differ appreciably from numerical estimates for g^* given by the lattice and Monte Carlo calculations [10-16]; such estimates are usually extracted from the data obtained for the linear (χ) and

non-linear (χ_4) susceptibilities related to each another via g_4 :

$$\chi_4 = \left. \frac{\partial^3 M}{\partial H^3} \right|_{H=0} = -\chi^2 m^{-2} g_4, \quad (8)$$

Since the convergence of the numerical estimates for g^* given by the resummed RG series is oscillatory, an account for higher-order (six-loop, seven-loop, etc.) terms in the expansion (3) will not avoid this discrepancy [9]. That is why we believe that it reflects the influence of the singular (non-analytical) contribution to the β -function.

The critical exponents for the Ising model ($n = 1$) and for those with $n = 0$ and $n = -1$ are estimated by the Padé-Borel summation of the five-loop expansions (4), (5) for γ^{-1} and η . Both the five-loop RG (Table 1) and the lattice (Table 2) estimates for g^* are used in the course of the critical exponent evaluation. To get an idea about an accuracy of the numerical results obtained the exponents are estimated using different Padé approximants, under various values of the shift parameter b , etc. In particular, the exponent η is estimated in two principally different ways: by direct summation of the series (5) and via the resummation of RG expansions for exponents

$$\eta^{(2)} = \frac{1}{\nu} + \eta - 2, \quad \eta^{(4)} = \frac{1}{\nu} - 2, \quad (9)$$

which possess a regular structure favouring the rapid convergence of the iteration procedure. The typical error bar thus found is about 0.05.

The results obtained are collected in Table 3. As is seen, for small exponent η and in some other cases the differences between the five-loop RG estimates and known exact values of the critical exponents exceed the error bar mentioned. Moreover, in the five-loop approximation the correction-to-scaling exponent ω of the 2D Ising model is found to be close to the value $4/3$ predicted by the conformal theory [17] and to the estimate 1.35 ± 0.25 extracted from the high-temperature expansions [18] but differs markedly from the exact value $\omega = 1$ [19] and contradicts to the conjecture $\omega = 2$ [20]. This may be considered as an argument in favour of the conclusion that non-analytical contributions are visible in two dimensions.

The field theory enables us also to find the higher-order, sextic coupling constant entering the free energy expansion in powers of the magnetization M . For 2D Ising model

$$F(M) - F(0) = m^2 \left[\frac{1}{2} \frac{M^2}{Z} + g_4 \left(\frac{M^2}{Z} \right)^2 + \sum_{k=3}^{\infty} g_{2k} \left(\frac{M^2}{Z} \right)^k \right], \quad (10)$$

where m is a renormalized mass, Z being a field renormalization constant. In the critical region, where fluctuations are so strong that they completely screen out the initial (bare)

interaction, the behaviour of the system becomes universal and dimensionless effective couplings g_{2k} approach their asymptotic limits g_{2k}^* .

In order to estimate g_6^* we calculate RG expansion for g_6 and then apply Pade-Borel-Leroy resummation technique to get proper numerical results. As is well known, accurate enough numerical estimates may be extracted only from sufficiently long RG series. We have obtained the expression for g_6 in the four-loop approximation [21] which turned out to provide fair numerical estimates for the quantity of interest.

The method of calculating the RG series we used in [21] is straightforward. Since in two dimensions higher-order bare couplings are irrelevant in RG sense, renormalized perturbative series for g_6 can be obtained from conventional Feynman graph expansion of this quantity in terms of the only bare coupling constant - quartic coupling λ . In its turn, λ may be expressed perturbatively as a function of renormalized dimensionless quartic coupling constant g_4 . Substituting corresponding power series for λ into original expansion we can obtain the RG series for g_6 . As was shown [22,23], the one-, two-, three- and four-loop contributions are formed by 1, 3, 16, and 94 one-particle irreducible Feynman graphs, respectively. Their calculation along with the renormalization procedure just described gives:

$$g_6 = \frac{36}{\pi} g_4^3 \left(1 - 3.2234882 g_4 + 14.957539 g_4^2 - 85.7810 g_4^3 \right). \quad (11)$$

This series may be used for estimation of the universal number g_6^* .

With the four-loop expansion in hand, we can construct three different Padé approximants: $[2/1]$, $[1/2]$, and $[0/3]$. To obtain proper approximation schemes, however, only diagonal $[L/L]$ and near-diagonal Pade approximants should be employed. That's why further we limit ourselves with approximants $[2/1]$ and $[1/2]$. Moreover, the diagonal Padé approximant $[1/1]$ is also dealt with although this corresponds, in fact, to the usage of the lower-order, three-loop RG approximation.

Since the Taylor expansion for the free energy contains as coefficients the ratios $R_{2k} = g_{2k}/g_4^{k-1}$ rather than the renormalized coupling constants themselves:

$$F(z) - F(0) = \frac{m^2}{g_4} \left(\frac{z^2}{2} + z^4 + R_6 z^6 + R_8 z^8 + \dots \right), \quad z^2 = \frac{g_4 M^2}{Z}, \quad (12)$$

we work with the RG series for R_6 . It is resummed in three different ways based on the Borel-Leroy transformation and the Pade approximants just mentioned. The Borel-Leroy integral is evaluated as a function of the parameter b under $g_4 = g_4^*$. For the fixed point

coordinate the value $g_4^* = 0.6124$ [24] is adopted which is believed to be the most accurate estimate for g_4^* available nowadays. The optimal value of b providing the fastest convergence of the iteration scheme is then determined. It is deduced from the condition that the Padé approximants employed should give, for $b = b_{opt}$, the values of R_6^* which are as close as possible to each other. Finally, the average over three estimates for R_6^* is found and claimed to be a numerical value of this universal ratio.

The results of our calculations are presented in Table 4. As one can see, for $b = 1.24$ all three working approximants lead to practically identical values of R_6^* . Hence, we conclude that for 2D Ising model at criticality

$$R_6^* = 2.94, \quad g_6^* = 1.10. \quad (13)$$

How close to their exact counterparts may these numbers be? To clear up this point let us discuss the sensitivity of numerical estimates given by RG expansion (11) to the type of resummation. The content of Table 4 implies that, among others, the results given by Padé approximant $[2/1]$ turn out to be most strongly dependent on the parameter b . This situation resembles that for 3D $O(n)$ -symmetric model where Padé approximants of $[L - 1/1]$ type for β -function and critical exponents lead to numerical estimates demonstrating appreciable variation with b while for diagonal and near-diagonal approximants the dependence of the results on the shift parameter is practically absent [1,3,25]. In our case, Padé approximants $[1/1]$ and $[1/2]$ may be referred to as generating such "stable" approximations for g_6^* . Since for b varying from 0 to 15 (i.e., for any reasonable b) the magnitude of g_6^* averaged over these two approximations remains within the segment (1.044, 1.142) it is hardly believed that the values (13) can differ from the exact ones by more than 5%.

Another way to judge how accurate our numerical results are is based on the comparison of the values of g_6^* given by four subsequent RG approximations available. While within the one-loop order we get $g_6^* = 2.633$ which is obviously very bad estimate, taking into account of higher-order RG contributions to g_6 improves the situation markedly. Indeed, two-, three-, and four-loop RG series when resummed by means of the Padé-Borel technique with use of "most stable" approximants $[0/1]$, $[1/1]$, and $[1/2]$ yield for g_6^* the values 0.981, 1.129, and 1.051, respectively. Since this set of numbers demonstrates an oscillatory convergence one may expect that the exact value of renormalized sextic coupling constant lies between the higher-order – three-loop and four-loop – estimates. It means that the deviation of numbers

(13) from the exact values would not exceed 0.05.

It is instructive to compare our estimates with those obtained by other methods. S.-Y. Zinn, S.-N. Lai, and M. E. Fisher analyzing high temperature series for various 2D Ising lattices found that $R_6^* = 2.943 \pm 0.007$ [26]; almost identical value was obtained in [27]. Our result for R_6^* is seen to be in a brilliant agreement with this number. Of course, practical coincidence of the lattice and four-loop RG estimates is occasional and can not be considered as a manifestation of extremely high accuracy of the methods discussed. The closeness of these estimates to each another, however, unambiguously demonstrates high power of both approaches. Moreover, such a closeness shed a light on the role of a singular contribution to g_6 which can not be found perturbatively: this contribution is seen to be numerically small.

It is interesting also to address the results given by another field-theoretical approach – the ϵ expansion. For the Ising systems three terms in the ϵ expansion for R_6 are known [28]:

$$R_6^* = 2\epsilon \left(1 - \frac{10}{27} \epsilon + 0.63795 \epsilon^2 \right). \quad (14)$$

Let us apply a simple Padé-Borel procedure to this series as a whole and to the series in brackets and then put $\epsilon = 2$. We find $R_6^* = 3.19$ and $R_6^* = 3.12$ respectively, i.e. the numbers which differ from our estimate by less than 9%. Keeping in mind lack of a small parameter these values of R_6^* may be referred to as consistent. Proper account for higher-order terms in the ϵ expansion for R_6 should make corresponding numerical estimates closer to those extracted from 2D RG and high-temperature series. Very good agreement between the first number (13) and the estimate $R_6^* = 2.95 \pm 0.03$ [27] obtained by matching of the ϵ expansion with the exact results known for $D = 1$ and $D = 0$ may be considered as an argument in favor of this belief. One can find more details in recent comprehensive review [29].

Along with the RG calculations at physical dimension and the ϵ expansion, some other field-theoretical approach may be employed to estimate the critical parameters of 2D Ising model. We mean the method of the pseudo- ϵ expansion (see Ref. 19 in [2]). Pseudo- ϵ expansions for the Wilson fixed point coordinate g^* and critical exponents can be easily derived from the RG series (3)-(5) using standart technique. They are as follows [30]:

$$g^* = \tau + 0.716173621\tau^2 + 0.095042867\tau^3 + 0.086080396\tau^4 - 0.204139\tau^5, \quad (15)$$

$$\gamma^{-1} = 1 - \frac{1}{3}\tau - 0.113701246\tau^2 + 0.024940678\tau^3 - 0.039896059\tau^4 + 0.0645212\tau^5, \quad (16)$$

$$\eta = 0.033966147\tau^2 + 0.046628762\tau^3 + 0.030925471\tau^4 + 0.0256843\tau^5. \quad (17)$$

Note that the higher-order terms in series (15) and (16) have small numerical coefficients and irregular signs. Smallness of these coefficients enables one to obtain accurate enough estimates for g^* and critical exponent γ without addressing the Borel- transformation-based resummation methods.

To demonstrate this, conventional Padé triangles originating from (15) and (16) under $\tau = 1$ are presented here (Tables 5 and 6). Since diagonal and near-diagonal Padé approximants are known to exhibit the best approximating properties, the numbers 1.751 and 1.837 from Table 5 should be referred to as most reliable estimates for g^* . Averaging over them, we obtain $g^* = 1.794$ which differs from the exact value $g^* = 1.75436$ [24] by 2%. As seen from Table 5, it is the five-loop approximation that provides so good numerical result; almost all lower-order approximations suffers from dangerous poles resulting in strongly scattered estimates. The same is true for the susceptibility exponent. Indeed, the numbers given by the main working approximants $[2/3]$ and $[3/2]$, as well as by approximant $[4/1]$, are almost coincide with each other and are close to the exact value $\gamma = 1.75$. In contrast, approximants $[2/2]$ and $[1/3]$, corresponding to the four-loop order, have dangerous poles which considerably affect the results.

Unfortunately, the pseudo- ϵ expansion technique turns out to be much less powerful when applied to estimate "small" critical exponent η . Both the direct summation of the expansion (17) and Padé resummation of the series for "big" exponents γ and ν lead to the numbers differing by 0.1 and even more from the exact value $\eta = 0.25$ [30]. To the contrary, the pseudo- ϵ expansion for the ratio $R_6 = g_6/g_4^2$

$$R_6 = 4\tau(1 - 0.409036\tau + 0.305883\tau^2 - 0.437676\tau^3) \quad (18)$$

demonstrates good Padé summability. It is clearly seen from Table 7 [30]. Averaging over two working approximants $[2/2]$ and $[3/1]$ gives the number $R_6 = 2.90$ which is close to earlier estimates $R_6 = 2.94$ [21], $R_6 = 2.95$ [27], $R_6 = 2.943$ [26], and to high-precision values $R_6 = 2.94294$ [24], $R_6 = 2.94238$ [29,31]. Usage of more advanced, Padé-Borel resummation technique shifts the pseudo- ϵ expansion estimate to $R_6 = 2.94$ [30] making it practically equal to just mentioned numbers.

The area where 2D $\lambda\phi^4$ field theory can be successfully applied is not limited by Ising-like and $O(n)$ -symmetric systems. The RG analysis of 2D cubic, MN, chiral, and weakly disordered models proves to be rather effective provided the higher-order – four- and five-loop – approximations are used [9,32-34]. In particular, many-loop RG calculations reproduce with high accuracy the exact results known for 2D anisotropic systems with n -vector order parameters. Detailed description of the situation may be found in [9,33,34].

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TABLE I: The Wilson fixed point coordinate for models with $n = 1$, $n = 0$ and $n = -1$ in four subsequent RG approximations and the final five-loop estimates for $g^*(n)$.

n	[1/1]	[2/1]	[2/2]	[3/2]	g^* , 5-loop
1	2.4246	1.7508	1.8453	1.8286	1.837 ± 0.03
0	2.5431	1.7587	1.8743	1.8402	1.86 ± 0.04
-1	2.6178	1.7353	1.8758	1.8278	1.85 ± 0.05

TABLE II: The Wilson fixed point coordinate g^* and critical exponent ω for $0 \leq n \leq 8$ obtained in the five-loop RG approximation. The values of g^* extracted from high-temperature (HT) [11,13] and strong coupling (SC) [12] expansions, found by Monte Carlo simulations (MC) [14,15], obtained by the constrained resummation of the ϵ -expansion for g^* (ϵ -exp.) [13], and given by corresponding $1/n$ -expansion ($1/n$ -exp.) [13] are also presented for comparison.

n	0	1	2	3	4	8
g^*						
RG, 5-loop	1.86(4)	1.837(30)	1.80(3)	1.75(2)	1.70(2) ($b = 1$)	1.52(1) ($b = 1$)
HT	1.679(3)	1.754(1)	1.81(1)	1.724(9)	1.655(16)	
MC		1.71(12)	1.76(3)	1.73(3)		
SC	1.673(8)	1.746(8)	1.81(2)	1.73(4)		
ϵ -exp.	1.69(7)	1.75(5)	1.79(3)	1.72(2)	1.64(2)	1.45(2)
$1/n$ -exp.				1.758	1.698	1.479
ω						
RG, 5-loop	1.31(3)	1.31(3)	1.32(3)	1.33(2)	1.37(3)	1.50(2)

TABLE III: Critical exponents for $n = 1$, $n = 0$, and $n = -1$ obtained via the Padé-Borel summation of the five-loop RG expansions for γ^{-1} and η . The known exact values of these exponents are presented for comparison.

n		g^*	γ	η	ν	α
1	RG	1.837	1.79	0.146	0.96	0.07
		1.754 (HT)	1.74	0.131	0.93	0.14
	exact		7/4 (1.75)	1/4 (0.25)	1	0
0	RG	1.86	1.45	0.128	0.77	0.45
		1.679 (HT)	1.40	0.101	0.74	0.52
	exact		43/32 (1.34375)	5/24 (0.20833)	3/4 (0.75)	1/2 (0.5)
-1	RG	1.85	1.18	0.082	0.62	0.76
		1.473 (SC)	1.15	0.049	0.59	0.82
	exact		37/32 (1.15625)	3/20 (0.15)	5/8 (0.625)	3/4 (0.75)

TABLE IV: The values of R_6^* obtained by means of the Padé-Borel-Leroy technique for various b within three-loop (approximant [1/1]) and four-loop (approximants [1/2] and [2/1]) RG approximations. The estimate for $b = 1$ in the middle line is absent because corresponding Padé approximant turns out to be spoilt by a positive axis pole.

b	0	1	1.24	2	3	4	5	7	10	15
[1/1]	2.741	2.908	2.937	3.009	3.077	3.125	3.161	3.212	3.258	3.301
[1/2]	2.827	-	2.936	2.877	2.853	2.838	2.828	2.814	2.800	2.787
[2/1]	3.270	2.988	2.936	2.800	2.667	2.568	2.491	2.380	2.273	2.171

TABLE V: The Wilson fixed point location g^* , extracted from pseudo- ϵ expansion (15) by means of constructing Padé approximants $[L/M]$. Coordinates of "dangerous" poles of Padé approximants, i. e. those lying on the real positive semiaxis are indicated as subscripts.

M L	1	2	3	4	5
0	1.000	1.716	1.811	1.897	1.693
1	3.523 _{1.4}	1.826 _{7.5}	2.724 _{1.1}	1.837	
2	1.425	1.918 _{3.0}	1.850 _{6.1}		
3	2.601 _{1.4}	1.751			
4	1.194				

TABLE VI: Numerical values of the critical exponent γ obtained by Padé summation of series (16) for γ^{-1} .

M L	0	1	2	3	4	5
0	1.000	1.500	1.808	1.730	1.859	1.660
1	1.333	2.024 _{2.9}	1.744	1.778	1.777	
2	1.558	1.702	1.800 _{5.2}	1.777		
3	1.646	6.871 _{1.1}	1.772			
4	1.732	1.718				
5	1.714 _{6.1}					

TABLE VII: Padé triangle for the universal ratio R_6 given by pseudo- ϵ expansion (18).

M L	1	2	3	4
0	4.000	2.364	3.587	1.837
1	2.839	3.064	2.867	
2	3.148 _{4.5}	2.940		
3	2.621			